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Mode-splitting behaviour of an asymmetric y-branching electron waveguide

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Abstract. A new configuration of electron waveguide, namely the y-branching electron waveguide, is proposed. Mode-splitting behaviour is revealed to be the result of symmetry of the structure and transverse mode of the waveguides. The transport properties of the structure are theoretically studied by applying the boundary-matching method. The effects of multireflection and resonance in the structure are demonstrated. Prospective applications are also suggested.

1. Introduction

With modern lithographic techniques, structures which confine the motion of electrons to channels of width as narrow as 10 nm or even narrower can be realized. It has been pointed out that electron behaviour in such channels is highly analogous to that of electromagnetic waves in waveguides [1, 2]. Therefore such a narrow channel is referred to as an electron waveguide. It is widely recognized that an electron waveguide will play an important role for the development of quantum devices [1, 2]. Recently, electron waveguides with various configurations and structures have been studied both theoretically and experimentally [3–10], and some of them have been proposed as the basis of quantum devices [2, 11–13]. In the present paper, we propose a new configuration of electron waveguide, i.e. an asymmetric y-branching electron waveguide, and study its transport properties theoretically. Mode-splitting behaviour of this kind of waveguide has been revealed and prospective applications are suggested. In integral optics, similar structures have been demonstrated [14, 15].

Figure 1 is the schematic illustration of the asymmetric y-branching waveguide. Two separate output waveguides (A and B) are connected to the input waveguide asymmetrically. Waveguide A is basically the extension of the input waveguide but with a little smaller width.

Assume that electrons with some energy which allows two modes in the input waveguide (waveguide 1) but only one in the waveguide A to be excited are launched in. It can be reasonably expected that electrons in mode 1 of the input waveguide will transmit easily into waveguide A since it sees weak scattering. For mode 2, however, the flux intensity which is independent on the transverse wavefunction must be zero at the middle of the waveguide, where the flux intensity for the excited mode of waveguide A should be close to its maximum. The continuity of flux requires that transmission of mode 2 to waveguide A must be small. Most of the current is thus forced to take other paths, namely either being transferred to waveguide B or reflected back to the input waveguide. Upon certain

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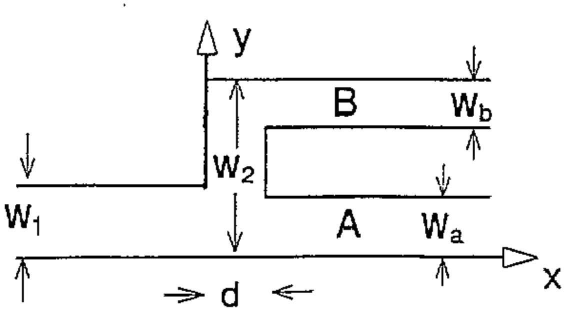

Figure 1. Schematic illustration of the asymmetric y-branching waveguide.

conditions, most of current carried by mode 2 may transmit to waveguide B and such a desired y-branching waveguide could serve as a mode splitter, i.e. basically mode 1 is transferred to waveguide A and mode 2 to waveguide B. Therefore the symmetry of the structure and the transverse mode of the device determine its transport behaviour.

2. Theoretical method

The boundary-matching method [6–8] is applied to obtain the electron wavefunction of the structure. The configuration shown in figure 1 is divided into three regions, i.e. $x < 0$, $0 < x < d$ and $x > d$. Assume that the electron with energy E_F is injected on the mth mode of the input waveguide. The wavefunction in region $x < 0$ can be expressed as

$$\Psi_1(x, y) = \phi_1^m(y) \exp\left(k_1^m x\right) + \sum_{n=1}^{N_1} I_n \phi_1^n(y) \exp\left(-ik_1^n x\right) \tag{1}$$

while, for $0 < x < d$,

$$\Psi_2(x, y) = \sum_{n=1}^{N_2} \left[II_n^+ \exp\left(ik_n^2 x\right) + II_n^- \exp\left(-ik_2^n x\right)\right] \phi_2^n(y) \tag{2}$$

and, for $x > d$,

$$\Psi_3(x, y) = \sum_{n=1}^{N_A} A_n \exp\left(ik_A^n x\right) \phi_A^n(y) + \sum_{n=1}^{N_B} B_n \exp\left(ik_B^n x\right) \phi_B^n(y) \tag{3}$$

where $\phi_i^n(y)$ is the nth transverse eigenfunction of waveguide i (i can be 1, 2, A or B). As shown in figure 1, waveguide 1 is the input waveguide; waveguide 2 is the middle waveguide with width W_2 and length d, and waveguides A and B are the output waveguides. On the assumption that the confinements of the waveguides are of the hard wall type, i.e. the potential beyond waveguides is infinite, $\phi_i^n(y)$ can be expressed in simple sine functions. k_i^n is the wavevector of the nth mode of waveguide i. Energy conservation gives

$$\hbar^2 k_i^n / 2m^* + E_i^n = E_F \tag{4}$$

where m^* is the effective mass of the electron and E_i^n the transverse eigenenergy of the nth mode of waveguide i. With the hard-wall assumption of the confinement, we obtain

$$E_i^n = \left(\hbar^2/2m^*\right)\left(n\pi/W_i\right)^2 \tag{5}$$

where W_i is the width of waveguide i. In equations (1)–(3), $\exp(ikx)$ represents a forward (from left to right in figure 1) wave and $\exp(-ikx)$ represents a backward wave. N_i is the number of modes of waveguide i included in the calculation.

For precise calculation, all the modes including not only the propagating modes whose transverse energies are lower than E_F but also the evanescent modes whose transverse energies are higher that E_F should be considered. Since the contribution from high evanescent modes is negligible, including a limited number of evanescent modes could meet satisfactory precision.

At $x = 0$, continuity of the wavefunction and its first derivative requires that

$$\Psi_1(x = 0, y) = \Psi_2(x = 0, y) \tag{6}$$

$$\left[\mathrm{d}\Psi_1(x, y)/\mathrm{d}x\right]\big|_{x=0} = \left[\mathrm{d}\Psi_2(x, y)/\mathrm{d}x\right]\big|_{x=0}. \tag{7}$$

Similarly, at $x = d$,

$$\Psi_2(x = d, y) = \Psi_3(x = d, y) \tag{8}$$

$$\left[\mathrm{d}\Psi_2(x, y)/\mathrm{d}x\right]\big|_{x=d} = \left[\mathrm{d}\Psi_3(x, y)/\mathrm{d}x\right]\big|_{x=d}. \tag{9}$$

In order to obtain the $N(N = N_1 + 2N_2 + N_A + N_B)$ coefficients in equations (1)–(3), i.e. I_n (n = 1, 2, ..., N_B), II_m^+ and II_m^- (m = 1, 2, ..., N_2), A_l (l = 1, 2, ..., N_A) and B_k (k = 1, 2, ..., N_B), one can multiply equations (6) and (8) by $\phi_2^m(y)^*$ (m = 1, 2, ..., N_2), equation (7) by $\phi_1^n(y)^*$ (n = 1, 2, ..., N_2), and equation (9) by $\phi_A^l(y)^*$ (l = 1, 2, ..., N_A) and $\phi_B^k(y)^*$ (k = 1, 2, ..., N_B), respectively, and integrate the resulting equations over y to yield N algebraic equations for the coefficients required. The solution of this equation group gives all the coefficients and hence the wavefunction.

The transmission coefficients are defined as

$$C_{1A}^{mj} = |A_j|^2 \tag{10}$$

$$C_{1B}^{mj} = |B_j|^2 \tag{11}$$

where C_{1A}^{mj} and C_{1B}^{mj} are the transmissions from the mth mode of waveguide 1 to the jth mode of waveguide A and B, respectively. The transmission probability [1, 2] of the current can be expressed as

$$T_{1A}^{mj} = C_{1A}^{mj} k_A^j / k_1^m \tag{12}$$

$$T_{1B}^{mj} = c_{1B}^{mj} k_B^j / k_1^m \tag{13}$$

where T_{1A}^{mj} and T_{1B}^{mj} are the probabilities of current transmission from the mth mode of waveguide 1 to the jth mode of waveguides A and B, respectively.

3. Results

The transmission probabilities versus W_2 are shown in figure 2. We use the normalized unit in the present paper. The length is in units of W which is chosen arbitrarily and, correspondingly, the wavevector is in units of π/W and energy $\hbar^2\pi^2/2m^*W$. For the present calculation, W_1 is chosen to be 1.2 and W_a 1.0. E_F is chosen to be 3.0 so that only one mode in waveguide 1 and two in waveguide A are excited. It can be seen from figure 2 that mode 1 of waveguide 1 transmits to waveguide A mostly. For mode 2, the transmission probability of waveguide A is very small but that of waveguide B oscillates with large amplitude. By taking the junctions of different waveguides as scattering centres, one at the lower intersection where three channels are connected together and the other at the upper corner, the oscillation can be considered to be the result of interference of the multiple reflected waves between these two scattering centres. Since the width in the x direction of the upward channel is chosen as $d = 1.0$, it can be shown that only one mode

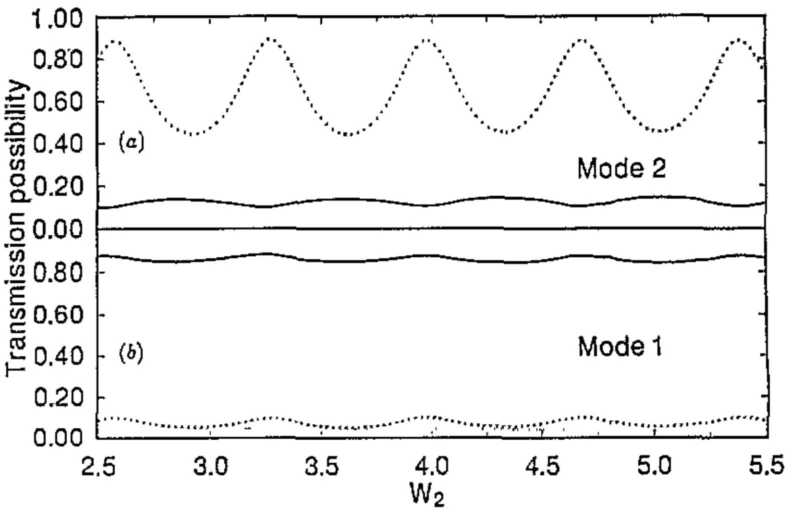

Figure 2. Transmission probability versus W_2: (a) ——, T_{1A}^{21}; ······, T_{1B}^{21}; (b) ——, T_{1A}^{11}; ······, T_{1B}^{11}. ($E_F = 3.0$; $W_1 = 1.2$; $d = 1.0$; $W_a = 1.0$; $W_b = 0.7$.)

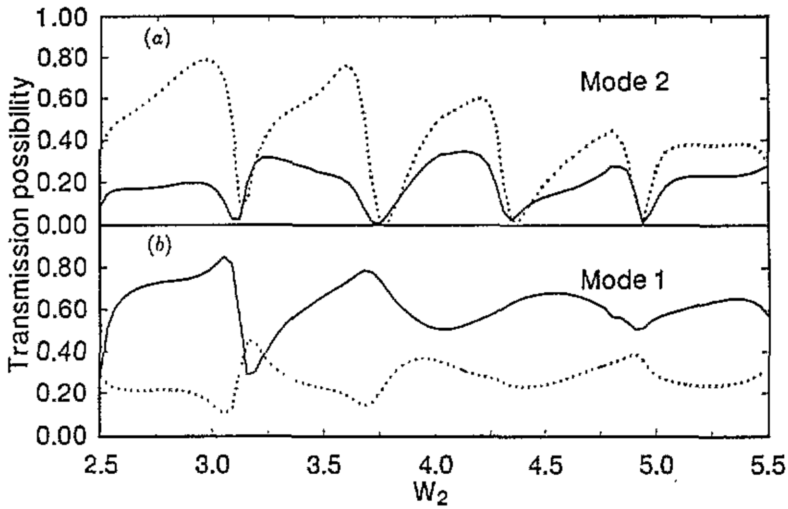

Figure 3. The same as figure 2 except that the width d is 1.5 of the upward channel.

is excited and propagates along the y direction; the oscillation is hence single periodic, as we see from the figure. The period ΔW_2 of the oscillation can be simply obtained from $2k\ \Delta W_2 = 2\pi$, where k is the wavevector of electron in the upward channel. With the parameters that we have, one obtains $\Delta W_2 = 0.71$, which is in good agreement with that shown in figure 2.

Figure 3 is the same as figure 2 except that $d = 1.5$. The oscillation becomes complicated as we expected since, in this case, two modes are excited in the upward channel and the resonance is no longer of a single-periodic nature. However, one can still choose W_2

carefully so that the current carried by mode 2 transmits to waveguide B mostly, i.e. one can design the asymmetric y-branching electron waveguide as a good mode splitter.

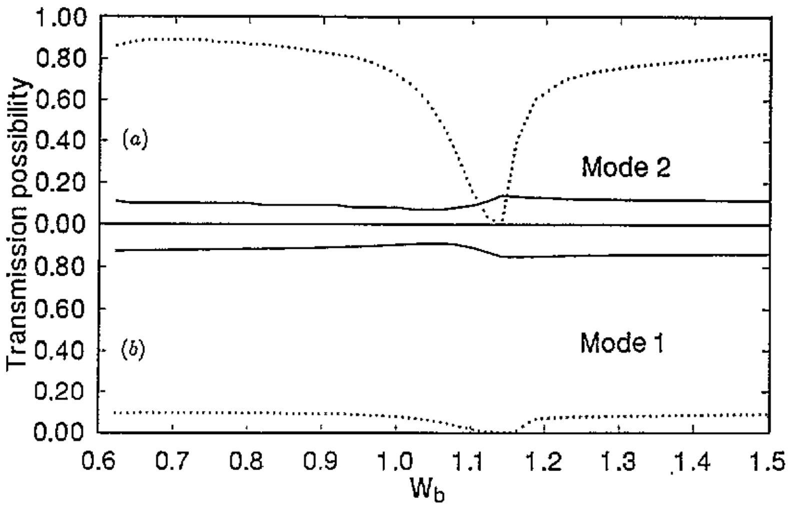

Figure 4. Transmission probability versus W_b: (a) ——, T_{1A}^{21}; ······, T_{1B}^{21}; (b) ——, T_{1A}^{11}; ······, T_{1B}^{11}. ($E_F = 3.0$; $W_1 = 1.2$; $W_2 = 3.3$; $d = 1.0$; $W_a = 1.0$.)

We have also studied the variation in the transmission probability with W_b, the width of waveguide B. The result is shown in figure 4. A clear dip appears at around $W_b = 1.15$. This could not be attributed to the multiple reflection as it is found to appear for any W_2. Detailed analysis indicates that the dip occurs when the electron energy is just below the threshold of the higher propagating mode. This type of rapid variation in transmission probability has been found and discussed in previous studies of bends in electron waveguides [16]. It is considered to be the result of the resonance of quasi-bound states in the bend. It is well known that in the bends of waveguides there exist bound states [16] below the threshold of the lowest mode (note the upper corner of the present structure is an 'L'-type bend). Also, there are quasi-bound states just below the thresholds of the higher subband. For the lower subbands, these quasi-bound states are resonance states. When the electron energy coincides with one of the quasi-bound states, electrons coupled strongly with them and the coupling causes a current dip [16]. Such a coupling should appear as a subband whose bottom just above the electron energy is becoming more involved since the presence of the higher subband is the main reason for the existence of the quasi-bound state. We calculated transmission coefficients C_{1B}^{21} and C_{1B}^{22} and showed the result in figure 5. It can be seen that as expected C_{1B}^{22} is much higher at $W_b \approx 1.15$. The transmission probability T_{1B}^{22} of the lowest mode thus decreases and a current dip appears.

If W_b increases further, mode 2 will be excited and carry a large part of the current. Figure 6 shows that, after $W_b > 1.2$, mode 2 carries a much larger part of the output current. This is in agreement with the mode conversion shown by Sols and Macucci [16]. Such variations in transmission probability will be repeated when more subbands are involved.

This asymmetric y-branching waveguide as a mode splitter might lead to a number of prospective applications. For instance, it can be developed as a switch if one applies a mode selector on the input waveguide [17]. By converting the input mode from mode 1 to mode 2, one can switch the current from waveguide A to waveguide B and vice versa.

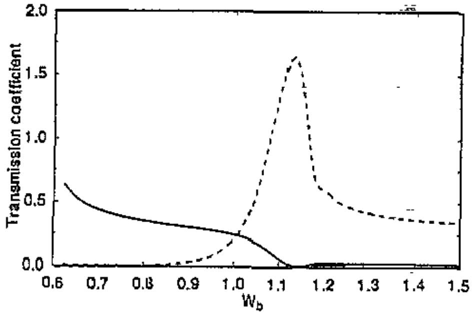

Figure 5. Transmission coefficients C_{1B}^{21} (——) and C_{1B}^{22} (- - - -) versus W_b. All the parameters are the same as those in figure 4.

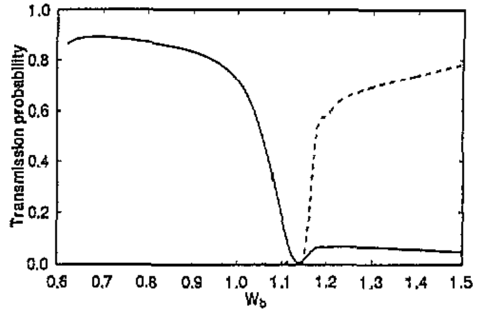

Figure 6. Transmission probabilities T_{1B}^{21} (——) and T_{1B}^{22} (- - - -) as a function of W_b. All the parameters are the same as those in figure 4.

In summary, an asymmetric y-branching waveguide has been theoretically studied. The mode-splitting behaviour and some other unusual transport properties are revealed. The results show that symmetry of the structure and transverse modes of waveguides is the major contribution to the mode-splitting behaviour. A rich structure of transmission probability is found and explained to be the results of multiple reflection and resonance in the structure. A current switch is proposed as an example of prospective applications. For simplicity, in the present paper all the waveguides are assumed to be along the x or the y direction. However, one could expect the mode-splitting behaviour not to be restricted to a particular configuration as it is a result of symmetry of the system. Investigation of the dependence of the mode-splitting effect on the shape of the branch is under way.

Acknowledgment

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